Question 1 (100 total points)

A. (40 points) Consider a version of the Lucas asset pricing model in which the representative agent maximizes:

$$E[\sum_{t=0}^{\infty} \beta^t \log(c_t)$$

There are two assets "trees" in the economy. Asset 1 yields a risky aggregate dividend d_t that follows the stochastic process:

$$d_{t+1} = d_t \varepsilon_{t+1}$$

where:

$$\varepsilon_t = \begin{cases} 1+\delta & \text{with probability } \pi \\ 1 & \text{with probability } 1-\pi \end{cases}$$

where $0 < \pi < 1$, $0 < \delta < 1/\beta - 1$ and d_0 is given. The second asset yields constant dividends each period equal to d_0.

(a) Define a recursive competitive equilibrium in this environment.

(b) Prove that the risk-free rate declines over time in this economy (in expectation or stochastically). Give the economic intuition/explanation for why.

(c) Solve for the limiting value of the risk-free rate. Give the economic explanation for why the limiting risk-free rate is less than $1/\beta$.

(d) Prove whether the ratio of the price of equity (the price of the tree) to its current dividend rises or falls over time and solve for its limiting value. Give the economic intuition/explanation for why it rises or falls over time.

B. (60 points) Consider the following model of unemployment. Agents have constant absolute risk aversion preferences over consumption c_t:

$$E_0 \left[-\frac{1}{\sigma} \sum_{t=0}^{\infty} \beta^t e^{-\sigma c_t} \right],$$

where $\sigma > 0$ and $0 < \beta < 1$. If a worker is unemployed, he finds a job with probability $p(a)$ where a is a *monetary* search expenditure cost. He also receives a benefit b, so his benefit net of search cost is $b - a$. If he finds a job, it pays a constant wage w for all time. (All jobs are identical and last forever.) The worker also has access to borrowing and lending in a risk-free asset with constant gross rate of return $R = 1 + r = 1/\beta$. Assume for simplicity that there is no limit or borrowing and that consumption may be negative.

(a) Consider first the case of an employed worker who begins a period with assets k. The law of motion for his asset stock is thus:

$$k' = Rk + w - c.$$

Find the worker's value function $V^e(k)$ and consumption $c^e(k)$.

(b) Now consider the case of an unemployed worker who receives a constant unemployment benefit b in each period. Write down his Bellman equation.

(c) Conjecture that search effort a is independent of k. Then conjecture that consumption when unemployed has the form $c^u(k) = rk + x$. Using the agent's Euler equation (with the results from (a)), find an expression for x.

(d) Guess and verify the unemployed agent's value function has the form:

$$V^u(k) = \frac{-e^{-\sigma(rk+x)}}{\sigma(1-\beta)}.$$

Can you verify the original conjecture that a is independent of k?

(e) Now consider the problem of optimal unemployment insurance, where a social planner chooses the amount of transfers b every period in order to provide an unemployed agent with a certain level of promised utility V in the most cost effective way. The planner does observe the agent's initial assets k but does not observe the search effort a or the division of assets between consumption and saving. Write the recursive cost minimization problem for $C(V, k)$: the minimized cost of providing utility V to an unemployed agent with assets k. [Note that there are no benefits or taxes on employed workers, so the benefits must simply be provided for the spell of unemployment.] Be explicit about incentive and promise-keeping constraints.

Question 2 (100 total points)

A. (50 points) Consider the following growth model. There is a representative household whose utility function is:

$$\sum_{t=0}^{\infty} \beta^t \left\{ \log(c_t) - \frac{1}{2} l_t^2 \right\}$$

where c_t is consumption and l_t is labor supply. The resource constraint is:

$$c_t + g_t = l_t$$

where g_t is government consumption, which is given by:

$$g_t = \begin{cases} 0 & \text{for } t \neq 10 \\ \bar{g} & \text{for } t = 10 \end{cases}$$

where $\bar{g} > 0$. The government takes the g_t sequence above as given and uses linear taxes on labor income and debt to finance it.

(i) Define an equilibrium for this model economy.

(ii) Formulate the Ramsey problem for the government.

(iii) Draw a time series plot of the optimal labor income tax rates for periods $t = 0$ through $t = 20$.

B. (50 points) Consider an endowment economy with two periods $t = 1, 2$ and two households $i = 1, 2$. Let c_t^i and y_t^i denote household i's consumption and endowment in period t, respectively. Households have the same utility function:

$$\log(c_1^i) + \log(c_2^i)$$

and their endowments are given by:

$$\begin{aligned} y_1^1 &= 3, \quad y_2^1 = 1; \\ y_1^2 &= 1, \quad y_2^2 = 3. \end{aligned}$$

Households are not allowed to borrow from others.

(i) Define an equilibrium for this model economy.

(ii) Calculate an equilibrium. Is the equilibrium allocation Pareto optimal?

Consider now the following fiscal policy intervention. In period $t = 1$, the government gives a lump sum transfer T_1 to each household and finances the transfer by issuing debt B_2. Then in period $t = 2$, the government repays the debt by collecting a lump sum tax T_2. Households can of course buy government bonds but they are still unable to borrow (or short sell government bonds), so their bond holdings b_2^i must be non-negative.

(iii) Define an equilibrium for this extended model economy.

(iv) Show that, by choosing the fiscal policy (T_1, T_2, B_2) appropriately, the government can implement an allocation that Pareto dominates the equilibrium allocation in (ii).