1. Consider a version of the neoclassical growth model with the following characteristics. There is a representative household with utility function

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

where c_t is consumption. The resource constraint is

$$c_t + k_{t+1} - (1-\delta)k_t + g \leq f(k_t), \quad \forall t$$
$$k_0 = \bar{k}$$

where k_t is capital and g is government spending.

The government takes g as given and uses taxes and debt to finance it. The tax instruments at its disposal are linear taxes on consumption, linear taxes on capital income, and lump sum taxes.

Answer the following questions and fully justify your answers. Make sure you explicitly state any assumptions about the model primitives that you use.

1. Define a competitive equilibrium. (20 points)

2. Consider the following scenario. At the beginning of period 0 the government commits to imposing constant tax rates on consumption and capital income. But then in period $T > 0$ it unexpectedly deviates from this commitment and makes a permanent increase in the capital income tax rate. What happens to consumption and capital before and after period T? Explain where you used the assumption that the tax rate increase was unexpected. (40 points)

3. Suppose now that the government faces the following two constraints: (i) it cannot use lump sum taxes, and (ii) it cannot tax capital income in period 0. What is the best fiscal policy under these constraints? Be sure to clearly state what you mean by "best." (40 points)

Question 2: 100 total points.

A. (50 points) Consider a version of the Lucas asset pricing model where the intertemporal discount factor is stochastic. That is, the representative agent has preferences:

$$E_0\left[\sum_{t=0}^{\infty} u(t, c_t)\right],$$

where

$$u(t, c_t) = \beta_t \log(c_t).$$

Let there be one "tree" and let the aggregate dividend d_t and the discount factor β_t follow a joint Markov process with transition function $F(d', \beta'|d, \beta)$. Note that the timing is such that at date t, the discount factor between dates t and $t+1$ is stochastic.

(a) Find the expressions (Euler equations) which determine the equilibrium price P_t of a claim to the endowment process, and R the risk free interest rate, and describe how they vary over time.

(b) Specialize to the case in which there are two states of nature $(d_1, \beta_1), (d_2, \beta_2)$, where $d_2 > d_1$ and $\beta_2 \neq \beta_1$. In addition, suppose that the states are i.i.d. and let π denote the probability $d_t = d_1$. Find an expression for the risk free interest rate.

(c) Continuing with the previous part, suppose instead that the discount factor is constant at $\beta = \pi\beta_1 + (1-\pi)\beta_2$. Under what conditions will the risk free rate in this case be lower than the one in the previous part where β is stochastic? Interpret your answer.

B. (50 points) When real balances enter the utility function, an agent seeks to maximize:

$$E_t \sum_{i=0}^{\infty} \beta^i U\left(C_{t+i}, m_{t+i}, L_{t+i}\right)$$

where L_t is leisure and $m_t = M_t/P_t$ is real balances, subject to the budget constraint in real terms:

$$w_t(1-L_t) + (r_t + 1 - \delta)\, K_{t-1} + (1+i_{t-1})\frac{P_{t-1}}{P_t} b_{t-1} + \frac{P_{t-1}}{P_t} m_{t-1} + \tau_t = C_t + K_t + b_t + m_t.$$

Here r_t is the real interest rate, i_t is nominal risk-free interest rate (on nominal bonds), b_t is real holdings of nominal bonds, and τ_t is lump sum taxes or transfers.

(a) Find the optimality conditions for the choice of leisure, consumption, and the holdings of capital, bonds, and money.

(b) The value of money in utility terms can be written as λ_t/P_t, where λ_t is the period-t Lagrange multiplier on the budget constraint, and is interpretable as the marginal utility of wealth. Find an expression for the value of money in terms of expected discounted future flows.

(c) From the optimality conditions, find three expressions (Euler equations) relating: (i) the intertemporal marginal rate of substitution and the real interest rate, (ii) the intertemporal marginal rate of substitution, the nominal interest rate, and inflation, and (iii) the marginal rate of substitution between consumption and real balances and the nominal interest rate.

(d) Suppose the utility function takes the form: $U(C_t, m_t, L_t) = u(C_t, L_t) + v(m_t)$. How do variations in the money supply, both in terms of its level and growth rate, affect the economy?

3) (100 points) Each agent goes through 3 periods of life: kid, young parent, old parent (and then dies). Each kid is born with a stochastic ability a. The distribution of the kid's ability is a function of the parent's ability, i.e. $a' \sim A(a'|a)$. Each young parent gives birth to one kid and makes decisions for him. A parent (or a grand-parent) cannot purchase insurance against the ability of his children.

The young parent invests e units of consumption goods and n units of time in his *own* human capital accumulation ("on-the-job training"), e_k units of consumption goods in his kid's human capital. Assume that the total amount of time endowed to each individual is 1 in each period. Human capital evolves according to

$$h'_o = a(nh)^{\gamma_1} e^{\gamma_2} + (1-\delta)h$$

$$h'_k = a_k(h)^{\gamma_1} e_k^{\gamma_2} + (1-\delta)h$$

where h is the human capital of the parent when he is a young adult, h'_o is the human capital of the parent when he gets old, and h'_k is the human capital of the child when he grows up and becomes a young parent.

The amount the young adult earns in the second period depends on the wage rate, w, the stock of human capital and a labor market "luck" shock ε.The young parent, for instance, earns the amount $wh(1-n)\varepsilon$. The luck shock ε is drawn from the distribution Φ and is iid across generations.

The young parent saves s for his old age when young, starts off the second period with inter-vivos transfers (transfers made while parent is alive) of i and begins the third period with bequests b from his parent. Assume that the young parent makes decisions *after* she receives her inter-vivos transfer from his parent and that the old parent makes decisions after he has receives the bequest b. Preferences are defined as

$$u(c_y) + \beta \mathbb{E} \left\{ u(c'_o) + \theta V' \right\},$$

where c_y is the consumption of the young parent, c'_o is the consumption of the old parent, and V' is the lifetime utility of her child after he grows up and the expectation operator is over future abilities. θ is the weight the parent puts on her child's utility. Assume that we are in a stationary equilibrium.

1. Let $V(\cdot)$ be the value for the young parent and $J(\cdot)$ for the old parent, formulate the Bellman equations for the young and old parent. Be careful to clarify the states and controls. (15 points)

2. Briefly outline an argument for why the pair of Bellman equations is a contraction. (10 points)

3. Briefly outline an argument that demonstrates that the value function is concave (Asssume that it exists and is given by the unique solution to the Bellaman's equation above). Also outline an argument that demonstrates that the Value function $V(\cdot)$ is differentiable. (10 points)

4. Derive all the first order conditions and the envelope conditions. Interpret the Euler equation(s). (20 points)

5. Compare the FOCs for e and e_k. Do they look different in structure? Why? (5 points)

6. Consider the General Equilibrium version of the model presented above. That is, apart from a continuum of consumers, we also have a standard Neoclassical firm that rents capital and human capital from consumers. Now suppose that markets are complete. What do the optimal choices of savings and investment in children look like? Explain how this differs from the incomplete market allocation. (10 points)

7. Imagine in this economy we considered a government that levies a tax on earnings (so that earnings for the young are $(1\text{-}\tau)wh(1-n)\varepsilon$, where τ is the tax rate) and rebates it back to (young and old) households in a lumpsum fashion. Would (all) consumers want this tax-transfer scheme starting from no governmental intervention whatsoever? Why? (10 points)

8. In the standard Aiyagari-Bewley model, the real interest rate in the steady state is less than the rate of time preference. Does the same result hold in this economy? Explain briefly why you think it holds or does not hold. What if we were to eliminate human capital from the model - how would your answer change? (10 points)

9. Consider an intergenerational redistribution scheme wherein a lump-sum tax is levied on levied on the old and the money is redistributed to the young in a lumpsum fashion. Which households are affected by this redistribution? Which households are unaffected? (10 points)