Econ 714: Handout 4 [1]

1 Phase diagrams[2]

Consider a version of the (deterministic) optimal growth model with government. There is an exogenous stream of government purchases $\{G_t\}$ that the planner takes as given. The household does not value government purchases, but they must be funded with real resourses. So the planner chooses the allocation of consumption c_t and capital k_{t+1} to maximize the household utility (over consumption, with labor supplied inelastically) subject to resource contraint:

$$k_{t+1} + c_t + G_t = (1 - \delta)k_t + f(k_t)$$

1. Suppose that government purchases are constant at $G_t = G$. How does the introduction of government spending affect the steady state levels of consumption and capital, relative to the case where $G = 0$?

2. Suppose that initially the economy is in a steady state with $G_t = G$, then there is a once-and-for-all unforeseen increase in purchases to a new higher level $G' > G$. What happens to consumption and capital immediately upon the impact of the change and in the long run?

3. Suppose that initially the economy is in a steady state with $G_t = G$, then at date T there is announcement that at the future date $T' > T$ purchases will increase to a new higher level $G' > G$ and remain there. What happens to consumption and capital at T, the date of the announcement? What happens between T and T'? What happens at T'?

2 Ramsey model[3]

Consider the following growth model. There is a representative household whose utility function is $\sum_{t=0}^{\infty} \beta^t (log(c_t) - \frac{1}{2} l_t^2)$, where c_t is consumption and l_t is labor supply. The resource constraint is: $c_t + g_t = l_t$, where g_t is government spending given by:

$$g_t = \begin{cases} 0 & \text{for } t \neq 10 \\ \bar{g} & \text{for} t = 10 \end{cases}$$

where $\bar{g} > 0$. The government takes the g_t sequence above as given and uses linear taxes on labor income and debt to finance it.

1. Define an equilibrium for this model economy.

2. Formulate the Ramsey problem for the government.

3. Draw a time series plot of the optimal labor income tax rates for period $t = 0$ through $t = 20$.

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[1] By Anton Babkin. This version: February 19, 2016.
[2] Fall 2014 problem set
[3] August 2013 prelim