HH

C_t : consump. , $\frac{M_t}{P_t}$: real money balances

leisure : $1 - N_t$

s.t. $$\max E_t \sum \beta^i \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b}\left(\frac{M_{t+i}}{P_{t+i}}\right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]$$

$$C_t = \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad \theta > 1$$

Step 1 :

$$\min_{c_{jt}} \int_0^1 P_{jt} c_{jt} \, dj$$

s.t. $$\left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \geq C_t$$

FOC : $$P_{jt} - \psi_t \left[\quad \right]^{\frac{1}{\theta-1}} c_{jt}^{-\frac{1}{\theta}} = 0$$

$$c_{jt} = \left(\frac{P_{jt}}{\psi_t}\right)^{-\theta} C_t$$

$$\Rightarrow \quad C_t = \left[\int_0^1 \left(\left(\frac{P_{jt}}{\psi_t}\right)^{-\theta} C_t \right)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$$

$$= \left(\frac{1}{\psi_t}\right)^{-\theta} \left[\int_0^1 P_{jt}^{1-\theta} dj \right]^{\frac{\theta}{\theta-1}} C_t$$

$$\Rightarrow \quad \psi_t = \left[\int_0^1 P_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \equiv P_t \rightarrow \text{price index}$$

$$\Rightarrow \quad c_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\theta} C_t$$

BC: $C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \left(\frac{W_t}{P_t}\right) N_t + \frac{M_{t-1}}{P_t} + (1+i_{t-1})\left(\frac{B_{t-1}}{P_t}\right) + \Pi_t$

$$L = E \sum \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b}\left(\frac{M_t}{P_t}\right)^{1-b} - \chi \frac{N_t^{1+\eta}}{1+\eta} \right)$$

$$- \sum \beta^t \lambda_t \left(C_t \; \cdots \; - \quad \right)$$

FOC: (C_t): $C_t^{-\sigma} = \lambda_t$

(M_t) $\gamma \left(\frac{M_t}{P_t}\right)^{-b} \cancel{\frac{1}{1-b}} = \lambda_t \left(\frac{1}{P_t}\right) - \beta \lambda_{t+1} \frac{1}{P_{t+1}}$

(B_t): $\lambda_t \frac{1}{P_t} = \lambda_{t+1} \beta \frac{(1+i_t)}{P_{t+1}}$

(N_t): $-\chi N_t^{\eta} + \lambda_t \left(\frac{W_t}{P_t}\right) = 0$

$\Rightarrow$ (1) $\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \frac{W_t}{P_t}$

(2) $C_t^{-\sigma} = \beta (1+i_t) E_t \left(\frac{P_t}{P_{t+1}}\right) C_{t+1}^{-\sigma}$

(3) $\frac{\gamma \left(\frac{M_t}{P_t}\right)^{-b}}{C_t^{-\sigma}} = \frac{i_t}{1+i_t}$

Firm:

$$C_{jt} = Z_t N_{jt} \qquad E(Z_t) = 1$$

① Cost minimization

$$\min_{N_{jt}} \left(\frac{W_t}{P_t}\right) N_{jt} + \varphi_t \left(C_{jt} - Z_t N_{jt} \right)$$

φ_t: marginal cost (real)

$$\frac{W_t}{P_t} = \varphi_t Z_t$$

Pricing Problem:

$$\Delta_{t,t+i} = \beta^i \left(\frac{C_{t+i}}{C_t}\right)^{-6}$$

$$E_t \sum w^i \Delta_{i,t+i} \left[\left(\frac{P_{jt}}{P_{t+i}}\right) C_{jt+i} - \varphi_{t+i} C_{jt+i} \right]$$

$$= E_t \sum w^i \Delta_{i,t+i} \left[\left(\frac{P_{jt}}{P_{t+i}}\right)^{1-\theta} - \varphi_{t+i} \left(\frac{P_{jt}}{P_{t+i}}\right)^{-\theta} \right] C_{t+i}$$

FOC:

$$E_t \sum w^i \Delta_{i,t+i} \left[(1-\theta) \frac{(P^*)^{-\theta}}{P_{t+i}^{1-\theta}} + \theta \varphi_{t+i} \frac{p^{*-\theta-1}}{(P_{t+i})^{-\theta}} \right] C_{t+i}$$

$$\|$$

$$\left[(1-\theta) \frac{p^*}{P_{t+i}} + \theta \varphi_{t+i} \right] \left(\frac{1}{p^*} \right) \left(\frac{P_t^*}{P_{t+i}} \right)^{-\theta} C_{t+i} = 0$$

$$\Rightarrow \frac{P_t^*}{P_t} = \frac{\theta}{\theta-1} \frac{E_t \sum w^i \beta^i C_{t+i}^{1-6} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta}}{E_t \sum w^i \beta^i C_{t+i}^{1-6} \left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}}$$

When $W=0$, prices are flexible

$$\frac{P_t^*}{P_t} = \left(\frac{\theta}{\theta-1}\right)\varphi_t \equiv \mathcal{M}\varphi_t \qquad \mathcal{M} = \frac{\theta}{\theta-1}$$

i.e. firm sets P_t^* to a markup $\mathcal{M}>1$ over nominal MC $P_t\varphi_t$

firms exante the same $\quad P_t^* = P_t \Rightarrow \varphi_t = \frac{1}{\mathcal{M}}$

$$\frac{Z_t}{\mathcal{M}} = \frac{W_t}{P_t} = \chi \frac{N_t^{\eta}}{C_t^{-6}}$$

→ approximate around steady state, let $\hat{x}_t$ be percentage change around steady state.

$$Z_t = \bar{Z} e^{\hat{Z}_t} \approx \bar{Z}(1+\hat{Z}_t)$$

$$\frac{\bar{Z} e^{\hat{Z}_t}}{\mathcal{M}} = \chi \frac{\bar{N}^{\eta} e^{\hat{N}_t \eta}}{\bar{C}^{-6} e^{-\hat{C}_t 6}}$$

$$1 + \hat{Z}_t = 1 + \hat{N}_t \eta + \hat{C}_t 6$$

$$\Rightarrow \quad \hat{Z}_t = \eta \hat{n}_t^f + 6 \hat{C}_t^f$$

also: $\quad y = n_t Z_t \quad \Rightarrow \quad \hat{x}_t^f = \hat{n}_t^f + \hat{Z}_t$

in eq'm $\quad \hat{y}_t^f = \hat{C}_t^f, \qquad \hat{y}_t^f = (1+\eta)\,\hat{n}_t^f + 6\hat{y}_t^f$

$$= \frac{1+\eta}{\eta}\hat{Z}_t - \frac{(1+\eta)6}{\eta} \hat{y}_t^f + 6\hat{y}_t^f$$

$$\hat{y}_t^f = \left(\frac{1+\eta}{6+\eta}\right)\hat{Z}_t$$

<u>Now</u> : when $w \neq 0$

$$P_t = \left(\int_0^1 P_{jt}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}$$

$$\Rightarrow \quad P_t^{1-\theta} = (1-w)(P_t^*)^{1-\theta} + w P_{t-1}^{1-\theta}$$

approximate.

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{k} \hat{e}_t \qquad \tilde{k} = \frac{(1-w)(1-\beta w)}{w}$$

(i)

$$1 = (1-w)\left(\frac{P_t^*}{P_t}\right)^{1-\theta} + w\left(\frac{P_{t-1}}{P_t}\right)^{1-\theta} \qquad Q_t = \frac{P_t^*}{P_t}$$

$$= (1-w) Q_t^{1-\theta} + w \pi_t^{\theta-1} \qquad \pi_t = \frac{P_t}{P_{t-1}}$$

$$= (1-w)\bar{Q} e^{q_t(1-\theta)} + w \bar{\pi} e^{\hat{\pi}_t(\theta-1)}$$

steady state $\bar{Q}=1$, $\bar{\pi}=1$

$$1 = (1-w)(\underset{1+}{\hat{q}_t}(1-\theta)) + w\,\underset{1+}{\hat{\pi}}(\theta-1)\,)$$

$$0 = (1-w)\hat{q}_t - w\hat{\pi}_t \quad \Rightarrow \quad \hat{q}_t = \left(\frac{w}{1-w}\right)\pi_t$$

(ii)

$$\left(E_t \sum w^i \beta^i \underbrace{C_{t+i}^{1-6}\left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}} \right) Q_t = \mu\left[E_t \sum w^i \beta^i C_{t+i}^{1-6} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta} \right]$$

$\boxed{\text{approximate}}$

LHS:

$$\left(E_t \sum w^i \beta^i \bar{C}^{1-6} \frac{\bar{P}^{\theta-1}}{\bar{P}^{\theta-1}} e^{\hat{C}_{t+i}(1-6) + \hat{P}_{t+i}(\theta-1) - \hat{P}_t} \right) \bar{Q} e^{\hat{q}_t}$$

$$1 + (1-6)\hat{C}_{t+i} + (\theta-1)(\hat{P}_{t+i} - \hat{P}_t) + \hat{q}_t$$

$$\frac{C^{1-6}}{1-w\beta} + \frac{C^{1-6}}{1-w\beta}\hat{q}_t + C^{1-6} \sum w^i \beta^i \left[(1-6) E_t \hat{C}_{t+i} + (\theta-1) E_t(\hat{P}_{t+i} - \hat{P}_t) \right.$$

<u>RHS</u> :

$$\mu\left[\frac{C^{1-\sigma}}{1-\omega\beta}\varphi + \varphi C^{1-\sigma}\sum \omega^i\beta^i\left[(1-\sigma)E_t\hat{C}_{t+i} + E_t\hat{\varphi}_{t+i} + \theta(E_t\hat{P}_{t+i}-\hat{P}_t)\right]\right]$$

$Q_t = 1 \quad \mu\varphi_t = 1$

$$\frac{\hat{q}_t}{1-\omega\beta} + \sum \omega^i\beta^i(\cdots\cdots) = \sum \omega^i\beta^i\begin{pmatrix}(1-\sigma)E_t\hat{C}_{t+i}\\ +E_t\hat{\varphi}_{t+i}\\ +\theta(E_t\hat{P}_{t+i}-\hat{P}_t)\end{pmatrix}$$

$$(1-\sigma)E_t\hat{C}_{t+i} + (\theta-1)(E_t\hat{P}_{t+i}-\hat{P}_t)$$

$$\Rightarrow \left(\frac{1}{1-\omega\beta}\right)\hat{q}_t = \sum \omega^i\beta^i\left(E_t\hat{\varphi}_{t+i} + E_t\hat{P}_{t+i} - \hat{P}_t\right)$$

$$\hat{q}_t + \hat{P}_t = (1-\omega\beta)\sum \omega^i\beta^i\left(E_t\hat{\varphi}_{t+i} + E_t\hat{P}_{t+i}\right)$$

$\downarrow$

opital nonial price.

$$\hat{q}_t + \hat{P}_t = (1-\omega\beta)(\hat{\varphi}_t + \hat{P}_t) + \underbrace{\overset{(1-\omega\beta)}{\sum_{i=1}} \omega^i\beta^i\left(E_t\hat{\varphi}_{t+i} + E_t\hat{P}_{t+i}\right)}_{\omega\beta(E_t\hat{q}_{t+1} + E_t\hat{P}_{t+1})}$$

$$\Rightarrow \hat{q}_t = (1-\omega\beta)\hat{\varphi}_t + \omega\beta\Big(E_t\hat{q}_{t+1} + \underbrace{E_t\hat{P}_{t+1} - \hat{P}_t}_{E_t\pi_{t+1}}\Big)$$

$(\ast)$

$$\frac{\omega}{1-\omega}\pi_t = (1-\omega\beta)\hat{\varphi}_t + \omega\beta\left(\frac{\omega}{1-\omega}E_t\pi_{t+1} + E_t\pi_{t+1}\right)$$

$$\Rightarrow \pi_t = \kappa\hat{\varphi}_t + \beta E_t\pi_{t+1}$$

In flexible price,

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \frac{W_t}{P_t}$$

$$\Rightarrow \quad \hat{w}_t - \hat{p}_t = \eta \hat{n}_t + \sigma \hat{y}_t$$

$$\hat{y}_t = \hat{n}_t + \hat{z}_t$$

$$\Rightarrow \quad \hat{\varphi}_t = \hat{w}_t - \hat{p}_t - \hat{z}_t = \hat{w}_t - \hat{p}_t - (\hat{y}_t - \hat{n}_t)$$

$$= \eta \hat{n}_t + \sigma \hat{y}_t - \hat{y}_t + \hat{n}_t$$

$$= \eta \hat{y}_t - \eta \hat{z}_t + \sigma \hat{y}_t - z_t$$

$$= (\eta + \sigma)\hat{y}_t - (1+\eta)\hat{z}_t$$

however : $\quad y_t^f = \dfrac{1+\eta}{\sigma+\eta}\hat{z}_t$

$$\hat{\varphi}_t = (\eta+\sigma)\hat{y}_t - (\sigma+\eta)\hat{y}_t^f = \gamma(\hat{y}_t - \hat{y}_t^f)$$

$$\Rightarrow \quad \pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$

where $\kappa = \gamma \tilde{\kappa}$

$x_t = (\hat{y}_t - y_t^f)$

from Euler eq'n:

$$C_t^{-\sigma} = \beta R_t E_t \left(\frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\sigma}$$

$$-\sigma \hat{y}_t = -\sigma E_t \hat{y}_{t+1} + (\hat{i}_t - E_t \pi_{t+1})$$

$$\Rightarrow \hat{y}_t = E_t \hat{y}_{t+1} - \left(\frac{1}{\sigma}\right)(\hat{i}_t - E_t \pi_{t+1})$$

$$\Rightarrow x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)(\hat{i}_t - E_t \pi_{t+1}) + \overbrace{\left(E_t y_{t+1}^f - \hat{y}_t^f \right)}^{u_t}$$